Math 10B with Professor Stankova
Quiz 6; Tuesday, 3/5/2019
Section #206; Time: 9:30 AM
GSI name: Roy Zhao   **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True   **FALSE** The PMF function f goes from subsets of $\mathbb{R}$ to $[0, 1]$.

   > **Solution:** The function f goes directly from $\mathbb{R}$ to $[0, 1]$, not subsets.

2. **TRUE**   False If x is not in the range of X and f is the PMF of X, then $f(x) = 0$.

   > **Solution:** $f(x) = P(X = x)$ and since x is not in the range, then $P(X = x) = 0$.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (6 points) I am playing a game where I roll a die over and over until I either roll a 6, or roll the die 3 times. Let X be the random variable for how many times I need to roll the die. Compute and draw the PMF of X. (Hint: Can you roll the die 4 times? Calculate the range of X first)

   > **Solution:** The game must end by the end of the 3rd round so the range of X is $\{1, 2, 3\}$. Then $P(X = 1) = \frac{1}{6}$ because the only way it ends is if we roll a 6. Then $P(X = 2) = \frac{5}{6} \cdot \frac{1}{6}$ because we need to first not roll a 6 then roll a 6. Finally, we have that $P(X = 3) = 1 - \frac{1}{6} - \frac{5}{6}\frac{1}{6} = \frac{25}{36}$ because that is if the game does not end in the first or second round. So the PMF is
   >
   > | x | 1 | 2 | 3 |
   > |---|---|---|---|
   > | $f(x)$ | $\frac{1}{6}$ | $\frac{5}{36}$ | $\frac{25}{36}$ |

   (b) (2 points) Let Y be the random variable that is 1 if the first die roll is a 6 and 0 otherwise. What is the PMF of Y?

   > **Solution:**
   > | x | 0 | 1 |
   > |---|---|---|
   > | $f(x)$ | $\frac{5}{6}$ | $\frac{1}{6}$ |

(c) (2 points) Are X and Y independent random variables?

Solution: No they are not. Intuitively if we know that $Y = 1$, then we know that we rolled a 6 so we know that the game ended and so $X = 1$. In math, this says that

$$P(X = 1, Y = 1) = \frac{1}{6} \neq P(X = 1)P(Y = 1) = \frac{1}{36}.$$